Managing Availability Improvement Efforts with Importance Measures and Optimization

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Abstract

The effective performance of repairable systems is critical to the success of all organizations, and availability measures are one of the two most common sets of measures used in evaluating the performance of such equipment. When availability performance is inadequate, engineers need a methodology for prioritizing availability improvement efforts. These efforts could include actions that reduce the occurrence of system failures, improve the execution of equipment maintenance, and/or add redundancy to the system. We define a set of availability importance measures and apply these measures to a general class of two-state repairable systems. Analysis of these measures for three examples yields insight into the prioritization of availability improvement efforts. In general, we show that focusing on reducing the occurrence of system failures provides greater benefit than increasing the speed of equipment repair. In the presentation, we describe the formulation and analysis of a set of three optimization models that capture the trade-offs between improving availability performance and the investments required to achieve this improvement.

1. Introduction

All industrial and military organizations depend upon the effective performance of repairable systems (production equipment, material handling equipment, vehicles, communication devices, etc.). The performance of repairable systems can be measured in several ways, and the use of mathematical modeling for the purpose of analyzing and/or optimizing repairable system performance has been studied extensively in the literature. One of the two most common classes of repairable system performance measures is availability measures (the class of cost measures is the other). These measures essentially capture the proportion of time that a system is operational (available for use).

Barlow and Proschan (1975) define four measures of availability performance: the availability function, limiting availability, the average availability function and limiting average availability. The availability function, $A(t)$, or point availability, captures the probability that the system is functioning at time t ($t \geq 0$). Limiting availability, A, if it exists, is the asymptotic (long-run, steady-state) value of the availability function. As with the majority of studies into the availability performance of repairable systems, our focus is on limiting availability. Regardless if A or some other availability measure is of interest, then there are two ways to improve repairable system performance: (1) reduce the occurrence of equipment failures, (2) improve the execution of maintenance actions.

We consider three repairable systems (RS1, RS2, RS3) that are required to operate on a continuous basis and have binary status – functioning (operating properly) or failed – at any point in time. Note that the individual components comprising these systems also have binary status. RS1 is a single-component system. For this component, the durations of successive operating intervals are independent and identically distributed (IID) exponential random variables with a constant failure rate of λ. Successive repair times are IID exponential random variables with a constant repair rate of μ. For this system,

$$A = \frac{\mu}{\lambda + \mu} \tag{1}$$

RS2 is comprised of m independent components connected in series. Each component is of the type that comprises RS1. Note that component i has a constant failure rate of λ_i and a constant repair rate of μ_i, $i = 1, 2, \dots, m$. For this system,

$$A = \prod_{i=1}^{m} \frac{\mu_i}{\lambda_i + \mu_i} \tag{2}$$

RS3 is comprised of m independent subsystems connected in series. Each subsystem i is comprised of n_i independent and identical components (of the type that comprise RS1) connected in parallel and having a constant failure rate of λ_i and a constant repair rate of μ_i, $i = 1, 2, \dots, m$. For this system,

$$A = \prod_{i=1}^{m} \left[1 - \left(1 - \frac{\mu_i}{\lambda_i + \mu_i} \right)^{n_i} \right] \tag{3}$$

2. Availability Importance

Depending on the system's reliability and maintainability parameter values (e.g. failure rate, repair rate, number of components) and the criticality of the mission performed by the system, the availability performance of the system may be inadequate. In such a case, the engineers responsible for designing and operating the system need to explore options for improving system performance. For the repairable systems of interest, we define two availability importance measures, similar to the Birnbaum (1969) reliability importance measure, that can serve as guidelines in developing an availability improvement strategy. Our first availability importance measure is $I_{\lambda,i}$, which denotes the marginal, relative improvement in limiting availability resulting from a decrease to the failure rate of component i, $i = 1, 2, \dots, m$. Therefore,

$$I_{\lambda,i} = \frac{1}{A}\left|\frac{\partial A}{\partial \lambda_i}\right| = \left|\frac{\partial \ln(A)}{\partial \lambda_i}\right| \tag{4}$$

Our second availability importance measure is $I_{\mu,i}$, which denotes the marginal, relative improvement in limiting availability resulting from an increase to the repair rate of component i, i = 1, 2, … , m. Therefore,

$$I_{\mu,i} = \frac{1}{A}\left(\frac{\partial A}{\partial \mu_i}\right) = \frac{\partial \ln(A)}{\partial \mu_i} \tag{5}$$

For RS1, there is only one component, so we denote the availability measures as I_λ and I_μ. Applying equations (4) and (5) yields:

$$I_\lambda = \frac{1}{\lambda + \mu} \tag{6}$$

$$I_\mu = \frac{\lambda}{\mu(\lambda + \mu)} \tag{7}$$

Typically, $\mu >> \lambda$. Therefore, decreasing the failure rate provides greater marginal benefit. For RS2, we use equations (4) and (5) to compute the availability importance measures for each component. As a result, we obtain:

$$I_{\lambda,i} = \frac{1}{(\lambda_i + \mu_i)} \tag{8}$$

$$I_{\mu,i} = \frac{\lambda_i}{\mu_i(\lambda_i + \mu_i)} \tag{9}$$

i = 1, 2, … , m. Again, λ_i / μ_i << 1. Therefore, the top priority in the availability improvement effort should be the failure rate of the component having the smallest $\lambda_i + \mu_i$. Since $\lambda_i << \mu_i$, the component having the smallest $\lambda_i + \mu_i$ is likely to be the component having the smallest repair rate. For RS3, application of equations (4) and (5) yields:

$$I_{\lambda,i} = \frac{\dfrac{n_i \mu_i}{\lambda_i(\lambda_i + \mu_i)}\left(\dfrac{\lambda_i}{\lambda_i + \mu_i}\right)^{n_i}}{1 - \left(\dfrac{\lambda_i}{\lambda_i + \mu_i}\right)^{n_i}} \tag{10}$$

$$I_{\mu,i} = \frac{\dfrac{n_i}{\lambda_i + \mu_i}\left(\dfrac{\lambda_i}{\lambda_i + \mu_i}\right)^{n_i}}{1 - \left(\dfrac{\lambda_i}{\lambda_i + \mu_i}\right)^{n_i}} \tag{11}$$

$i = 1, 2, \ldots, m$. For a given subsystem, reducing the failure rate is more important than increasing the repair rate. However, determining the most important subsystem requires numerical analysis. See Table 1 for an example of RS3. In this example, the top priority for availability improvement should be reducing the failure rate of the components that comprise subsystem 10.

Subsystem (i)	n_i	λ_i	μ_i	$I_{\lambda,i}$
1	2	0.10	5.0	0.00754
2	3	0.30	7.5	0.00055
3	2	0.18	3.2	0.02992
4	4	0.55	6.0	0.00033
5	3	0.40	1.9	0.03276
6	2	0.23	4.3	0.02133
7	5	0.85	6.7	0.00009
8	4	0.65	3.3	0.00377
9	3	0.50	7.8	0.00123
10	1	0.10	9.5	0.10417

Table 1. RS3 Example

References

Barlow, R., Proschan, F. (1975). Statistical Theory of Reliability and Life Testing: Probability Models. New York: Rinehart and Winston.

Birnbaum, Z. (1969). On the importance of different components in multi-component systems. *Multi-Variate Analysis II*, pp. 581-592.